Vector exchanges in production of light meson pairs and elementary atoms.

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Abstract. The production of pseudoscalar and scalar mesons pairs and bound states (positronium or pionium atoms) in high energy $\gamma\gamma$ collisions at high energies provided by photon or vector meson exchanges are considered. The vector exchanges lead to nondecreasing with energy cross section of binary process $\gamma + \gamma \to h_a + h_b$ with h_a, h_b states in the fragmentation regions of initial particles. The production of light mesons pairs $\pi\pi, \eta\eta, \eta'\eta', \sigma\sigma$ as well as a pairs of positronium Ps and pionium A_π atoms in peripheral kinematics are discussed. Unlike the photon exchange the vector meson exchange needs a reggeization, leading to fall with energy. Nevertheless due to peripheral kinematics out of very forward production angles the vector meson exchanges dominated.

The proposed approach allows to express the matrix elements of the considered processes through impact factors, which can be calculated in perturbation models like Chiral Perturbation Theory (ChPT) or Nambu-Jona-Lasinio (NJL) model or determined from $\gamma\gamma$ sub-processes or vector mesons radiative decay widths.

We obtain the cross sections for pionium atom production in collisions of high energy pions and electrons with protons. The possibility to measure these processes in experiment are discussed.

1. Introduction

The next large project after LHC should be likely a linear e^+e^- accelerator at energy $\sqrt{s} = 0.5 - 1TeV$, giving exciting challenge to study $\gamma\gamma$ interactions at energies of hundreds GeV. The technology of obtaining the beams of high energy photons is based on the backward Compton scattering of laser light on high energy electrons [1], idea known for many years [2, 3].

Exclusive processes with hadronic final states test various model calculations and hadron production mechanisms. So far the meson pairs production in two photon collisions are measured [4, 5, 6] at $\gamma\gamma$ center of mass energy $W \leq 4$ GeV and scattering angle $|cos\theta| < 0.8$. In this work we investigate the production of light mesons pairs and elementary atoms (positronium Ps and pionium A_π atoms) in high energy $\gamma\gamma$ collisions in peripheral kinematics:

$$\gamma(k_1) + \gamma(k_2) \to h_a(p_1) + h_b(p_2); \quad h_a, h_b = \pi, \eta, \eta\prime, \sigma, Ps, A_\pi \tag{1}$$

Due to peripheral kinematics ($s = (k_1 + k_2)^2, t = q^2 = (p_1 - k_1)^2; \quad s >> |q^2|$) the created objects h_a, h_b have energies approximately equal to the energies of colliding photons and move along the directions of initial particles motion (center of mass of initial particles implied).

The dominant contribution to peripheral processes comes from large orbital momenta in scattering amplitude expansion. The background from low orbital momentum in peripheral kinematics is strongly suppressed unlike the processes allowing production at any angles. A typical example is the Born-term amplitude (π exchange in the t channel) of the process $\gamma\gamma \to \pi^+\pi^-$ whose differential cross section at small angles has additional suppression due to wide phase volume of the final state.

The another remarkable property of the relevant cross sections-they become independent from center of mass energy s of colliding particles starting from some threshold energy $\sqrt{s} \sim 2 - 3GeV$. The nondecreasing feature of pairs yield is a result of vector nature of the interaction (photon or vector meson exchanges in the t-channel (Fig.1)).

In peripheral kinematics one can use the perturbation models of hadrons like Chiral Perturbation Theory [7, 8] (ChPT) or Nambu-Jona-Lasinio [9] (NJL) model to describe the sub-processes at the relevant vertexes. One can expresses the matrix element of reaction (1) through so called impact factors, which are nothing else than the matrix elements of sub-processes (Fig.1a): $\gamma(k_1) + \gamma^*(q) \to h_a$ and $\gamma(k_2) + \gamma^*(q) \to h_b$ or (Fig. 1b): $\gamma(k_1) + V(q) \to h_a$ and $\gamma(k_2) + V(q) \to h_b$.

Let us briefly discuss the connection of matrix element of reaction (1) with relevant impact factors M^a, M^b (the details can be found in [10]).

According to the general rules the matrix element of the process (1) reads :

$$M = \frac{J^a_\rho J^b_\sigma}{q^2 - m_V^2} g^{\rho\sigma}, \tag{2}$$

$J^{a,b}$ are currents associated with blocks a, b of relevant Feynman diagram.

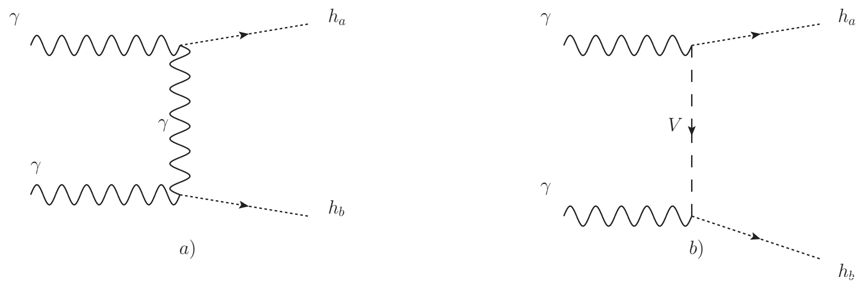

Figure 1. (a) The photon exchange in the process $\gamma + \gamma \to h_a + h_b$; (b) Exchange by vector meson.

Making use the infinite momentum frame parametrization of the transferred momentum:

$$q = \alpha k_1 + \beta k_2 + q_\perp; \quad q_\perp k_1 = q_\perp k_2 = 0; \; q_\perp^2 = -\vec{q}^2 < 0; \; k_1^2 = k_2^2 = 0. \quad (3)$$

and written metric tensor in the Gribov's form:

$$g^{\rho\sigma} = g_\perp^{\rho\sigma} + \frac{2}{s}(k_1^\rho k_2^\sigma + k_2^\rho k_1^\sigma). \quad (4)$$

one obtains the connection of matrix element of process (1) with impact factors (with power accuracy):

$$M = \frac{2s}{q^2 - m_V^2} M^a M^b; \quad M^a = \frac{J_\mu^a k_2^\mu}{s}; \quad M^b = \frac{J_\nu^b k_1^\nu}{s}. \quad (5)$$

The impact factors M^a, M^b don't decrease with energy and can be described in terms of perturbation strong interaction models like Nambu-Jona-Lasinio model or Chiral Perturbation Theory. The cross section of the processes (1) is connected with matrix element (5) in the standard way:

$$d\sigma^{ab \to h_a h_b} = \frac{1}{8s} \sum |M|^2 d\Gamma \quad (6)$$

Expressing the phase volume of the two final particles $d\Gamma$ through the Sudakov parameters (3) one can rewritten the two particles phase space volume:

$$d\Gamma = (2\pi)^4 \delta(k_1 + k_2 - p_1 - p_2) \frac{d^3 p_1}{2(2\pi)^3 E_1} \frac{d^3 p_2}{2(2\pi)^3 E_2} \quad (7)$$

in the following form [10]:

$$d\Gamma = \frac{d^2 q}{2(2\pi)^2 s} \quad (8)$$

As a result the differential cross section of the processes (1) reads:

$$d\sigma^{ab \to h_a h_b} = \frac{d^2 q}{(4\pi)^2 (\vec{q}^2 + m_V^2)^2} \sum_{spins} |M^a|^2 \sum_{spins} |M^b|^2 \quad (9)$$

Thus the knowledge of relevant impact factors allows one to calculate the cross sections of processes (1).

2. Mesons production. Photon exchange.

We start with the production of $\pi^0\pi^0$ pair in $\gamma\gamma$ collisions with photon exchange in the t-channel (Fig. 1a). The current algebra gives for the matrix element of neutral pion decay to two photons $\pi^0(p) \to \gamma(k_1, e_1) + \gamma(k_2, e_2)$:

$$M(\pi^0 \to \gamma\gamma) = \frac{\alpha}{\pi f_\pi}(k_1 e_1 k_2 e_2), \tag{10}$$

where $(abcd) = a^\alpha b^\beta c^\gamma d^\delta \epsilon_{\alpha\beta\gamma\delta}$ and $k_i, e_i(k_i)$ are the momenta and polarization vectors of real photons, $\alpha = \frac{e^2}{4\pi} = 1/137$ is the fine structure constant and $f_\pi = 92.2MeV$ is the pion decay constant measured in the $\pi^+ \to \mu^+\nu_\mu$ decay rate.
The pion radiative decay width is given by the textbook formula:

$$\Gamma(\pi^0 \to 2\gamma) = (\frac{m_\pi}{4\pi})^3(\frac{\alpha}{f_\pi})^2 = 7.76eV \tag{11}$$

The decay amplitude (10) can be used as impact factor in $\pi^0\pi^0$ production.
More elaborated impact factors considering the photon virtuality can be obtained if one calculates the triangle fermion loop with the light u and d quarks as a fermions [9]. Quarks charges and number of colors result in a factor $3((2/3)^2 - (1/3)^2) = 1$. After standard procedure of denominators joining, calculating the relevant trace in the fermions spin indices and integration over the loop momenta we obtain:

$$M(\pi^0 \to \gamma\gamma^*) = \frac{\alpha}{2\pi f_\pi}|\,[\vec{e},\vec{q}]\,|F_\pi(z), \quad z = \frac{\vec{q}^2}{m_q^2}$$

$$F_\pi(z) = N_\pi \int\limits_0^1 dx \int\limits_0^1 \frac{y dy}{1 - \rho_\pi^2 y^2 x(1-x) + zy(1-y)x}, \quad \rho_\pi = \frac{m_\pi}{m_q}. \tag{12}$$

Here m_q is the constituent quark mass which we put $m_q = m_u = m_d \approx 280MeV$, whereas N_π is the normalization constant $F_\pi(0) = 1$.
The similar expression for the sub-process of the scalar meson decay $\sigma \to \gamma\gamma^*$ reads:

$$M(\sigma \to \gamma\gamma^*) = \frac{5\alpha}{6\pi f_\sigma}|\,(\vec{e},\vec{q})\,|F_\sigma(z); \quad F_\sigma(0) = 1, \quad f_\sigma \approx f_\pi$$

$$F_\sigma(z) = N_\sigma \int\limits_0^1 dx \int\limits_0^1 \frac{y(1-4y^2x(1-x))dy}{1 - \rho_\sigma^2 y^2 x(1-x) + zy(1-y)x}, \quad \rho_\sigma = \frac{m_\sigma}{m_q}. \tag{13}$$

The combination of quark charges and color factor give a coefficient $3((2/3)^2 + (1/3)^2) = 5/3$. The nontrivial difference in numerators of (12) and (13) is a result of scalar nature of σ meson. As to the decay $\eta(\eta') \to \gamma\gamma^*$ it is enough to do the relevant replacements of masses in equation (12).
The amplitudes $M(\pi^0 \to \gamma\gamma^*), \quad M(\sigma \to \gamma\gamma^*)$ are nothing else than impact factors, one needs to calculate the cross sections of neutral mesons pairs production . Now we are in position to estimate the influence of the photon virtuality on the cross section of the reaction $\gamma\gamma \to \pi^0\pi^0$ from (1). Making use the relation:

$$\int\limits_0^{2\pi} \frac{d\phi}{2\pi}[\vec{q}\vec{e}_1]^2[\vec{q}\vec{e}_2]^2 = \frac{(\vec{q}^2)^2}{8}(1 + 2\cos^2\phi_0), \tag{14}$$

with ϕ_0 the azimuthal angle between the initial photons polarization vectors. Substituting expression (12) for π^0 impact factor in (9) we get:

$$\frac{d\sigma}{dz} = \frac{m_q^2}{8\pi}(\frac{\alpha}{4\pi f_\pi})^4\frac{(zF_\pi(z))^4}{(1+z^2)^2}(1+2cos^2\phi_0). \tag{15}$$

In the case of pions production one can safely neglect the small term $y^2x(1-x)m_\pi^2/m_q^2 < 0.05$ in the denominator of (12) with the result:

$$F_\pi(z) = 2\int\limits_0^1 dx \int\limits_0^1 \frac{ydy}{1+zy(1-y)x} = \frac{4}{z}\ln^2\left(\sqrt{1+\frac{z}{4}}+\sqrt{\frac{z}{4}}\right). \tag{16}$$

The total cross section of the two neutral pions production:

$$\sigma^{\gamma\gamma\to\pi_0\pi_0} = \sigma_0(1+2\cos^2\phi_0)I, \quad \sigma_0 = \frac{\alpha^4 m_q^2}{2^7\pi^5 f_\pi^4} \approx 2,6\times 10^{-2}pb,$$

$$I = \frac{1}{4}\int\limits_0^\infty \frac{dz}{z^4}\ln^8(\sqrt{1+z}+\sqrt{z}) = 0.3557. \tag{17}$$

Thus the expressions (9), (12), (13) allow to calculate the yields of any combination of light meson pairs produced in $\gamma\gamma$ collisions.

3. Bound states production

The considered approach is especially efficient in investigation of bound states formation in collisions of high energy particles. As a typical examples we examine the production of simplest atoms being the bound state of two charged pions (pionium atom A_π) and atom constructed from two fermions (positronium atom Ps).
To determine the pionium impact factor $\gamma\gamma^* \to A_\pi$ we take advantage of well known QED amplitude [11] for the process $\gamma(k_1,e_1)+\gamma^*(q)\to\pi^-(q_-)+\pi^+(q_+)$:

$$M^{\gamma\gamma^*\to\pi\pi} = \frac{4\pi\alpha}{s}[\frac{(2q_-e_1)((-2q_++q)k_2)}{2q_-k_1}+\frac{(-2q_+e_1)((2q_--q)k_2)}{-2q_+k_1}-2(e_1k_2)] \tag{18}$$

Account on that in atom pions have almost the same velocity $q_+ = q_- = p/2$; $2(pk_1) = 4m_\pi^2+\vec{q}^2$ and expressing p, e through the Sudakov variables:

$$p = \alpha_p k_2 + \beta_p k_1 + q_\perp; \quad e = \beta_e k_1 + e_\perp, \tag{19}$$

Making use the relation:

$$(pe_1)((p-q)k_2 - (pk_1)(ek_2) = -2s(\vec{q}\vec{e}_1) \tag{20}$$

the amplitude of two pions production with the same velocities takes the form:

$$M^{\gamma\gamma^*\to\pi\pi} = \frac{8\pi\alpha(\vec{e}\vec{q})}{\vec{q}^2+4m_\pi^2} \tag{21}$$

In order to obtain the amplitude for pionium production we use the relation [12] allowing to connect the amplitude of two free scalar mesons production with the production amplitude of their bound state A_π ‡

$$M^{\gamma\gamma^*\to A_\pi} = M^{\gamma\gamma^*\to\pi\pi}\frac{i\Psi(\vec{r}=0)}{\sqrt{m_\pi}} \tag{22}$$

Finally for the amplitude of pionium production in $\gamma\gamma^*$ collisions we get:

$$M^{\gamma\gamma^*\to A_\pi} = \frac{8i\pi\alpha(\vec{e}\vec{q})}{4m_\pi^2+\vec{q}^2}\frac{\Psi(0)}{\sqrt{m_\pi}}, \tag{23}$$

To obtain the impact factor for para-positronium creation $\gamma(k_1,e_1)+\gamma^*(q)\to Ps(p)$ we take advantage of the receipt [13, 14] of passage from free e^+e^- pair to their bound state and textbook expression [11] for e^+e^- pair creation in $\gamma\gamma$ collisions. As a result the matrix element for bound state creation takes the form:

$$M^{\gamma\gamma*\to Ps} = i\frac{4\pi\alpha}{s}\frac{m_e\sqrt{\alpha^3}}{\sqrt{4\pi}}\frac{1}{4}Tr[\hat{e}_1\frac{\hat{q}_- - \hat{k}_1+m_e}{(q_- - k_1)^2-m_e^2}\hat{k}_2$$
$$+\hat{k}_2\frac{-\hat{q}_+ + \hat{k}_1+m_e}{(-q_+ + k_1)^2-m_e^2}\hat{e}_2](\hat{p}+m_{Ps})\gamma_5. \tag{24}$$

Making use the relations $q+k_1=p=q_++q_-$ and $q_+=q_-=p/2$ one obtains:

$$M^{\gamma\gamma^*\to Ps} = \frac{4im_e\sqrt{\pi\alpha^5}}{4m_e^2+\vec{q}^2}|[\vec{e}_1,\vec{q}]|. \tag{25}$$

With the help of equation (9) and impact factors (23), (25) one can calculates the differential cross section of elementary atoms creation in the processes:

$$\gamma+\gamma\to S_1+S_2; \quad S_1,S_2=A_\pi, Ps. \tag{26}$$

For reader convenience and rough estimates of the order of total cross sections of bound state production by photon exchange mechanism (Fig.1a) we cite the expressions for the total cross sections relevant to reactions (26):

$$\sigma^{\gamma\gamma\to PsPs} = \frac{\pi\alpha^8}{96}r_e^2(1+2\cos^2\phi_0); \quad \sigma^{\gamma\gamma\to A_\pi A_\pi} = (\frac{r_e}{4r_\pi})^2\sigma^{\gamma\gamma\to PsPs};$$
$$\sigma^{\gamma\gamma\to PsA_\pi} = \frac{\pi\alpha^8}{64}r_\pi^2(3-2\cos^2\phi_0); \quad \sigma^{\gamma\gamma\to\pi^0 Ps} = \frac{\alpha^7}{32\pi^2 f_\pi^2}(1+2\cos^2\phi_0);$$
$$r_e=\frac{\alpha}{m_e}, r_\pi=\frac{\alpha}{m_\pi}. \tag{27}$$

Rough estimates of these cross sections show that they are really very small quantity of order $10^{-8}nb$.

4. Vector meson exchange in pairs production

Up to now we considered production processes provided by photon exchanges (Fig.1a). From the other hand the vector meson (Fig. 1b) exchanges also give nondecreasing with energy

‡ The square of pionium ground state wave function at origin has the form: $|\Psi(0)|^2 = \frac{\alpha^3 m^3}{8\pi}$

contribution to the processes (1). The problem with such type exchanges is connected with the fact that the Born approximation depicted on Fig. 1b badly violated for strong interactions. To take into account the higher order contributions of strong interaction one would replaces the exchanged vector meson propagator in (9) by its reggeized analog [15]

$$\frac{1}{t-m_V^2} \to \alpha' \frac{1-e^{-i\pi\alpha(t)}}{2} \Gamma(1-\alpha(t)) (\frac{s}{s_0})^{\alpha(t)}, \tag{28}$$

where $\alpha(t)$ is the Regge trajectory of vector meson

$$\alpha(t) = \alpha(0) + \alpha' t \tag{29}$$

The Γ function contains the pole propagator $sin^{-1}(\pi\alpha(t))$ and in the limit $t \to m_V^2$ the expression (28) reduced to the standard pole propagator. The detailed characteristics of Regge trajectories of different vector mesons can be found in work [16] and references therein.
Later on for estimation of vector mesons contribution to the relevant cross sections we use the simplified suppression factor:

$$R(s,t) = (\frac{s}{s_0})^{2(\alpha(t)-1)} \approx \frac{s_0}{s}; \;\; s_0 \approx 1GeV^2. \tag{30}$$

The impact factors corresponding to the vector mesons exchanges depend on the considered process and would be obtained as it has been done above for photon exchanges.
As an example let us consider the process of two charged pions production $\gamma\gamma \to \pi^+\pi^-$ for which the photon exchange is absent. The main contribution to this reaction at high energies gives the ρ exchange.§ The matrix element of radiative decay of charged meson $\rho^+(p,e_1) \to \pi^+(p_\pi) + \gamma(k,e_2)$ reads $M = g_+(pe_1ke_2)$, where the constant g_+ can be determined from the relevant decay width :

$$\Gamma^{\rho^+\to\pi^+\gamma} = \frac{g_+^2}{96\pi} (\frac{m_\rho^2 - m_\pi^2}{m_\rho})^3. \tag{31}$$

Comparing this relation with the experimental value of the $\rho^+ \to \pi^+\gamma$ branching ratio [18] $B = 4.5 \times 10^{-4}$, $\Gamma = 67keV$ one gets $g_+ \approx 0.21GeV^{-1}$.
In the case when one of the photons is virtual it is enough to do the simple replacement $g_+ \to g_+F(z)$ with

$$F(z) = \frac{4}{z} \ln^2(\sqrt{1+\frac{z}{4}} + \sqrt{\frac{z}{4}}); \quad z = \frac{\vec{q}^2}{m_q^2}. \tag{32}$$

The differential cross section of the process $\gamma\gamma* \to \pi^+\pi^-$ in peripheral kinematic takes the form

$$\begin{aligned} d\sigma &= \frac{d\vec{q}^2 d\phi}{32\pi^2} \frac{|M^{(1)}|^2 |M^{(2)}|^2}{(\vec{q}^2 + m_\rho^2)^2}, \\ M^{(1)} &= \frac{g_+}{2}[\vec{q}\vec{e}_1]F(z); \quad M^{(2)} = \frac{g_+}{2}[\vec{q}\vec{e}_2]F(z). \end{aligned} \tag{33}$$

§ The pion exchange [17] dominates only at small transfer momenta $t \le 4m_\pi^2 \le 0.1GeV^2$ and fall off with energy much stronger than vector exchanges.

Averaging over azimuthal angle according to equation (14) for the total cross section we obtain:

$$\sigma(\gamma\gamma \to \pi^+\pi^-) = \frac{g_+^4 m_q^2}{32\pi}(1 + 2\cos^2\phi_0)I;$$
$$I = \int_0^\infty \frac{dz}{z^2(z + (\frac{m_\rho}{2m_q})^2)^2} \ln^8(\sqrt{1 + \frac{z}{4}} + \sqrt{\frac{z}{4}}) \approx 0.372$$
$$\sigma^{\gamma\gamma\to\pi^+\pi^-} \approx 60(1 + 2\cos^2\phi_0)(\frac{s_0}{s})nb. \tag{34}$$

In the same way one can estimates the contribution from ρ, ω exchanges to the process of two neutral pions production $\gamma\gamma \to \pi^0\pi^0$, determining the constants g_ρ, g_ω from experimental data on the relevant decay rates [18]

$$\Gamma(\rho^0 \to \pi^0\gamma) = 8.9 \times 10^{-5}GeV; \quad g_\rho = 0.25GeV^{-1}$$
$$\Gamma(\omega \to \pi^0\gamma) \ = 70 \times 10^{-5}GeV; \quad g_\omega = 0.71GeV^{-1}. \tag{35}$$

For the total cross section of the process $\gamma\gamma \to \pi^0\pi^0$ provided by vector meson exchanges we obtain:

$$\sigma^{\gamma\gamma\to\pi^0\pi^0} = 3(1 + 2\cos^2\phi_0)(\frac{s_0}{s})\mu b. \tag{36}$$

5. Pionium atom production in ep and πp collisions.

In recent years there has been a significant effort to extract the $\pi\pi$ s-wave scattering lengths a_I with total isospin I=0, 2 from experimental data on pionium atom A_π creation. The scattering lengths determination with high precision allows to check the predictions of low-energy hadron theories such as Chiral Perturbation Theory (CHPT) or Nambu-Jona-Lasinio model (NJL) which give it value with unprecedented for strong interaction accuracy $\sim 2\%$ [19].
The main goal of experiment DIRAC [20] at PS CERN has been the determination of pions scattering lengths difference $a_0 - a_2$ from the measurement of pionium atom lifetime, which is connected with this difference by the relation [21]:

$$\Gamma = \frac{1}{\tau} = \frac{2}{9}\sqrt{\frac{2(m_{\pi^+} - m_{\pi^0})}{m_\pi}}(a_0^0 - a_0^2)^2 m_\pi^3 \alpha^3. \tag{37}$$

At present due to experiment Dirac and experiments on kaons decays [22, 23] the scattering lengths determined from experimental data with precision comparable with theoretical predictions.
Below we will consider the peripheral mechanism of creation of two charged pions in collision of high energy electron with the proton and similar one with the initial high energy negatively charged π-meson instead electron

$$e(p_1) + p(p_2) \to e(p_1') + A_\pi(p) + p(p_2') \tag{38}$$

$$\pi(p_1) + p(p_2) \to \pi(p_1') + A_\pi(p) + p(p_2') \tag{39}$$

$s = (p_1 + p_2)^2 >> m_p^2$ with m_p a proton mass.
For the case of electron-proton collision the pion pair is created in the collision of virtual

photon emitted by electron and virtual ρ (ω) meson emitted by proton (Fig. 2a). In the case of π-meson proton interaction the pion pair is produced by two virtual ρ mesons (Fig. 2b). The

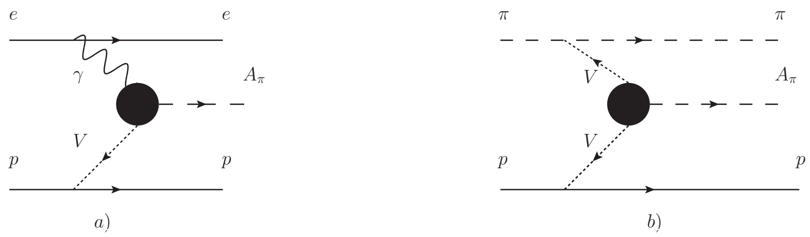

Figure 2. a) The pionium electroproduction in the process $e+p\to e+p+A_\pi$; b) Pionium production by pions $\pi+p\to\pi+p+A_\pi$

matrix element corresponding to these processes has the form:

$$M=\frac{G}{(q_1^2-m_1^2)(q_2^2-m_2^2)}J_1(p_1)_{\mu_1}T_{\mu\nu}J_p(p_2)_{\nu_1}G^{\mu\mu_1}G^{\nu\nu_1}, \tag{40}$$

with G is the product of the relevant coupling constants, $m_{1,2}$-masses of the exchanged vector particles; J_1, J_p are the currents connected with the colliding particles; tensor $T_{\mu\nu}$ describes the conversion of two vector mesons to pion pair.
The main contribution in peripheral kinematics (non-vanishing in limit $s\to\infty$) arises from relevant Green functions:

$$G^{\mu\mu_1}=\frac{2}{s}p_2^{\mu}p_1^{\mu_1};G^{\nu\nu_1}=\frac{2}{s}p_2^{\nu}p_1^{\nu_1}. \tag{41}$$

Matrix element of the sub-process of creation of pion pair with equal 4-momenta by two virtual vector particles

$$V_\mu(q_1)+V_\nu(q_2)\to\pi^+(q)+\pi^-(q) \tag{42}$$

is described by the tensor:

$$T_{\mu\nu}=\frac{2}{D}[q_{2\mu}q_{1\nu}+Dg_{\mu\nu}],D=-\frac{1}{2}[4m^2+\vec{q}_1^{\,2}+\vec{q}_2^{\,2}], \tag{43}$$

Combining these expressions one gets for the matrix element of the process $e+p\to e'+p'+A_\pi$:

$$M^{ep\to epA_\pi}=\frac{4s}{\vec{q}_1^{\,2}+m_e^2\beta_1^2}\frac{G_e}{\vec{q}_2^{\,2}+m_V^2}\Phi_e\Phi_A\Phi_p\Psi(0), \tag{44}$$

where $G_e=4\pi\alpha g_\pi g_p$, (g_π, g_p-coupling constants of ρ-meson with pion and proton, which we put $g_\pi=g_\rho=3$)

$$\Phi_e=\frac{1}{s}\bar{u}(p_1')\hat{p}_2u(p_1);\quad \Phi_A=\frac{1}{s}p_1^{\mu}p_2^{\nu}T_{\mu\nu}=-2\frac{\vec{q}_1\vec{q}_2}{D};$$
$$\Phi_p=\frac{1}{s}\bar{u}(p_2')\Gamma_\mu u(p_1)p_2^{\mu},\Gamma_\mu=\gamma_\mu F_1+\frac{1}{4M_p}(\hat{q}_2\gamma_\mu-\gamma_\mu\hat{q}_2)F_2, \tag{45}$$

Here $F_1=F_1(q_2^2), F_2=F_2(q_2^2)$ are Dirac and Pauli form-factors of proton.
The phase volume of the three particles in the final state:

$$d\Gamma=\frac{(2\pi)^4}{(2\pi)^9}\frac{d^3p_1'}{2E_1'}\frac{d^3p_2'}{2E_2'}\frac{d^3p_A}{2E_A}\delta^4(p_1+p_2-p_1'-p_2'-p_A), \tag{46}$$

can be reduced using the Sudakov variables to the following form:

$$d\Gamma = \frac{1}{(2\pi)^5}\frac{1}{4s}\frac{d\beta_1}{\beta_1}d^2\vec{q}_1 d\vec{q}_2. \tag{47}$$

Making use the summed over spin states of the squares of matrix elements of the relevant sub-processes:

$$\sum|\Phi_e|^2 = 2; \quad \sum|\Phi_p|^2 = 2[F_1^2 + \frac{\vec{q}_2^2}{4m_p^2}F_2^2];$$
$$|\Phi_A|^2 = \frac{4(\vec{q}_1\vec{q}_2)^2}{(4m^2+\vec{q}_1^2+\vec{q}_2^2)^2}, \tag{48}$$

where m is the pion mass. The cross section of the process $e+p\to e+p+A_\pi$ takes the form:

$$d\sigma^{ep\to epA_\pi} = \frac{\alpha^5 g_\pi^2 g_p^2}{2\pi^2}\frac{m^2\vec{q}_1^2 d\vec{q}_1^2\vec{q}_2^2 d\vec{q}_2^2}{(4m^2+\vec{q}_1^2+\vec{q}_2^2)^2(\vec{q}_1^2+m_e^2\beta_1^2)^2(\vec{q}_2^2+m_\rho^2)^2}$$
$$\times\ [F_1^2+\frac{\vec{q}_2^2}{4m_p^2}F_2^2]\frac{d\beta_1(1-\beta_1)}{\beta_1}; \quad \frac{4m^2}{s}<\beta_1<1. \tag{49}$$

Similar expression for the cross section with initial π meson instead of the electron:

$$d\sigma^{\pi p\to \pi pA_\pi} = \frac{\alpha^3 g_\pi^6 g_p^2}{64\pi^4}\frac{m^2\vec{q}_1^2 d\vec{q}_1^2\vec{q}_2^2 d\vec{q}_2^2}{(4m^2+\vec{q}_1^2+\vec{q}_2^2)^2(\vec{q}_1^2+m_\rho^2)^2(\vec{q}_2^2+m_\rho^2)^2}$$
$$\times\ [F_1^2+\frac{\vec{q}_2^2}{4m_p^2}F_2^2]\frac{d\beta_1(1-\beta_1)}{\beta_1}. \tag{50}$$

Integrating these expressions over phase volume one obtains the total yield of pionium atom. In the case of the electroproduction:

$$\sigma(ep\to epA_\pi) = \sigma_e D_e, \;\; \sigma_e = \frac{\alpha^5 g_\pi^2 g_p^2 m^2}{2\pi^2 m_\rho^4}\approx 0.3pb;$$
$$D_e = J_N[l_m^2 + l_\pi(l_m-1)-2], J_N = \int\limits_0^\infty \frac{xN^2dx}{(x+4)^2(x+N)^2}\approx 0.845;$$
$$l_m = \ln\frac{s}{4m^2}, \quad l_\pi = \ln\frac{m^2}{m_e^2}. \tag{51}$$

For $s = 100GeV^2$ the cross section $\sigma(ep\to epA_\pi)\approx 30pb$ is too small to be measured at present accelerators.
As to the pionium production by pions we obtain:

$$\sigma(\pi p\to\pi pA_\pi) = \sigma_\pi D_\pi, \quad \sigma_\pi = \frac{\alpha^3 g^8 m^2}{64\pi^2 m_\rho^4}\approx 217nb;$$
$$D_\pi = (l_m-1)I, I = \int\limits_0^\infty\int\limits_0^\infty \frac{x_1x_2dx_1dx_2}{(x_1+x_2)^2(x_1+1)^2(x_2+1)^2}\approx 0.133. \tag{52}$$

The total cross section turns out to be of the order $\sigma(\pi p\to\pi pA_\pi)\approx 178nb$ for s=80 GeV^2 (IHEP, Protvino) and thus can be measured at modern facilities.

In conclusion we note that the contribution from the channels with exchange of two photons is of order

$$\sigma_e^{\gamma\gamma} = \sigma_0 D_e^{\gamma\gamma}; \;\; \sigma_\pi^{\gamma\gamma} = \sigma_0 D_\pi^{\gamma\gamma}, \sigma_0 = \frac{8\alpha^7}{m^2} \approx 1,8 \times 10^{-3} pb. \tag{53}$$

In spite of a rather large enhancement factors $D_e^{\gamma\gamma} \sim 10 D_\pi^{\gamma\gamma} \sim 10^2$ the relevant contributions can be safely neglected.

6. The vector meson exchange reggeization

As was mentioned above the consideration of hadronic processes in peripheral kinematics in Born approximation is non-adequate. The effect of converting the ordinary vector mesons to the relevant Regge poles must be taken into account. It results in an additional suppression factor to the total cross sections of processes (38), (39):

$$R = \left(\frac{s_1 s_2}{s_0^2}\right)^{2(\alpha(0)-1)}, \tag{54}$$

Keeping in mind the kinematical relation $s_1 s_2 \approx 4m^2 s$ and puting $\alpha(0) \approx 0.5$:

$$R \approx \frac{s_0^2}{4sm^2}. \tag{55}$$

For instance at $s = 80 GeV^2$ it results in the suppression factor

$$R \approx 0.16. \tag{56}$$

So the realistic cross section for this energies is about $\sigma^\pi \approx 28 nb$.
Let us note that in the double pomeron exchanges in the process (39) (or pionium photoproduction off pomeron in the case of reaction (38)) such suppression factor is absent and at enough high energies the pomeron exchanges dominated. It is useful to estimate the energies from which the photon exchange becomes comparable with vector mesons one. For instance to obtain the matrix element for pionium electroproduction by two photon exchanges from the matrix element with one vector meson exchange (fig.2a) it is enough to do a simple replacement:

$$g_\pi g_p \frac{s_0}{2m\sqrt{s}} \to 4\pi\alpha \tag{57}$$

Thus only from energies $s \sim 10^5 GeV^2$ the contribution with two photon exchanges in pionium electroproduction becomes larger than the one with vector meson exchange.

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